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Dilaton-fixed scalar correlators and $AdS_5 \times S^5$-SYM correspondence

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Abstract: We address the question of AdS/CFT correspondence in the case of the 3-point function $\langle O_4 O_4 O_8 \rangle$. O_4 and O_8 are particular primary states represented by $\mathrm{tr}\, F^2 + \cdots$ and $\mathrm{tr}\, F^4 + \cdots$ operators in $\mathcal{N} = 4$ SYM theory and dilaton ϕ and massive "fixed" scalar ν in $D = 5$ supergravity. While the value of $\langle O_4 O_4 O_8 \rangle$ computed in large N weakly coupled SYM theory is non-vanishing, the $D = 5$ action of type IIB supergravity compactified on S^5 does not contain $\phi\phi\nu$ coupling and thus the corresponding correlator seems to vanish on the AdS_5 side. This is in obvious contradiction with various arguments suggesting non-renormalization of 2- and 3-point functions of states from short multiplets and implying agreement between the supergravity and SYM expressions for them. We propose a natural resolution of this paradox which emphasizes the 10-dimensional nature of the correspondence. The basic idea is to treat the constant mode of the dilaton as a part of the full S^5 Kaluza-Klein family of dilaton modes. This leads to a non-zero result for the $\langle O_4 O_4 O_8 \rangle$ correlator on the supergravity side.

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1. Introduction

Recent studies of 2- and 3-point functions of operators in short multiplets of $\mathcal{N} = 4$ super Yang-Mills theory in the context of AdS/CFT correspondence [1, 2, 3] (see [4] for a review) suggest that they are not renormalized in large-N limit [2],[5]–[12].[1] Since the supergravity correlator is interpreted as the large-N strong coupling SYM result, this implies that the exact SYM expressions for the large-N limit of the 2- and

[1] For other papers on correlators in the AdS/CFT correspondence see [13, 14] and references in [4].

3-point correlators of such operators should not contain non-trivial functions of 't Hooft coupling (the coordinate dependence of the $n \leq 3$ correlators is fixed uniquely by conformal invariance). This conclusion is supported by explicit SYM perturbative calculations [15, 16, 17, 18], general arguments based on $\mathcal{N} = 4$ superspace [17] or $\mathrm{U}(1)_Y$ symmetry [19] (or non-renormalization of anomalies [20]). However, one easily finds an apparent contradiction.

Consider the following $\mathrm{SU}(N)$ $\mathcal{N} = 4$ SYM operators

$$O_4 = \mathrm{tr}(F_{\mu\nu}F_{\mu\nu}) + \cdots , \tag{1.1}$$

$$\begin{aligned} O_8 &= \frac{3}{2}\,\mathrm{Str}\left[F^4 - \frac{1}{4}(F^2)^2\right] + \cdots \\ &= \mathrm{tr}\bigg(F_{\mu\nu}F_{\rho\nu}F_{\mu\lambda}F_{\rho\lambda} + \frac{1}{2}F_{\mu\nu}F_{\rho\nu}F_{\rho\lambda}F_{\mu\lambda} - \\ &\qquad - \frac{1}{4}F_{\mu\nu}F_{\mu\nu}F_{\rho\lambda}F_{\rho\lambda} - \frac{1}{8}F_{\mu\nu}F_{\rho\lambda}F_{\mu\nu}F_{\rho\lambda}\bigg) + \cdots , \end{aligned} \tag{1.2}$$

where dots stand for scalar and spinor terms which are required by $\mathcal{N} = 4$ supersymmetry and Str is the symmetrized trace of $\mathrm{SU}(N)$ generators as matrices in the fundamental representation. O_4 and O_8 are supersymmetric descendants of chiral primary operators $\mathrm{tr}(XX)$ and $\mathrm{tr}(XXXX)$, respectively,[2]

$$O_4 = (Q^4 + \bar{Q}^4)\,\mathrm{tr}\,X^2 , \qquad O_8 = Q^4\bar{Q}^4\,\mathrm{tr}\,X^4 . \tag{1.3}$$

For all the operators $\mathrm{tr}\,X^n$ the precise agreement between the (weighted) 3-point correlators computed in the large-N weakly coupled SYM and the AdS_5 supergravity was demonstrated in [10], so it seems natural to expect the same for their supersymmetry descendants.

Using the AdS/CFT correspondence recipe, one finds — either from the supermultiplet considerations [3, 21] or from the structure of the Born-Infeld action for a D3-brane in a curved background [22] — that on the supergravity side O_4 corresponds to the 5-D dilaton ϕ [23, 24], while O_8 corresponds to the fixed scalar ν [25].[3] The latter enters the $D = 10$ Einstein-frame metric as

$$ds^2_{10E} = e^{-\frac{10}{3}\nu(x)} g_{5mn}(x)dx^m dx^n + e^{2\nu(x)} d\Omega_5^2 . \tag{1.4}$$

The specific powers of e^ν in the metric are needed to decouple ν from the 5-d graviton; this ansatz generalizes to non-linear level the graviton mode decomposition considered in [27] where this massive ($m^2 = 32/R^2$) singlet scalar ν was identified with the

[2] The precise definition for these chiral primary operators is [10]: $C^I_{i_1\cdots i_k}\,\mathrm{tr}(X^{i_1}, \ldots, X^{i_k})$, where C^I is a totally symmetric traceless rank k tensor of $SO(6)$ and X^i are scalars of $\mathcal{N} = 4$ SYM theory. The action of supercharges in $\mathrm{tr}\,X^2$ and $\mathrm{tr}\,X^4$ is indicated only schematically. In particular, in O_8 one needs to specify the ordering of Q's and $\bar{Q}$'s to resolve the associated total derivative ambiguity.

[3] A relation between fixed scalars and F^4-type operators was earlier mentioned in [26] in the case of the D5+D1 system.

zero mode (S^5 independent part) of the trace of the perturbation of the metric of S^5. Note that the zero mode ν does not mix with the 4-form potential, while higher Kaluza-Klein modes do [27]. It is the lowest-mass member of the KK family of scalars with masses $m^2 = (k+4)(k+8)/R^2$, $k = 0, 1, \ldots$, and belongs to a separate massive $D = 5$ supermultiplet interacting with the $D = 5$, $\mathcal{N} = 8$ supergravity multiplet. Dimensionally reducing the dilaton-graviton sector of type IIB supergravity action to 5 dimensions, one easily finds [28] that the action does *not* contain $\phi\phi\nu$ coupling (in agreement with the possibility to perform the so-called consistent truncation of the full $D = 5$ theory to the $D = 5$ gauged supergravity subsector). This implies that on the AdS supergravity side

$$\langle O_4 O_4 O_8 \rangle_{\rm AdS} = 0\,.$$

However, the explicit computation of $\langle O_4 O_4 O_8 \rangle$ in the SYM theory performed below to the leading order in large-N expansion yields a non-vanishing result for this correlator.[4]

One then faces the following three options:

(i) There may be some subtlety in applying the nonrenormalization theorem for the 3-point functions of chiral primary operators $\mathrm{tr}\, X^n$ to their supersymmetry descendants and the correlator $\langle O_4 O_4 O_8 \rangle$ may actually depend on gauge coupling in such a way that it vanishes in the limit $\lambda \equiv g^2_{\rm YM} N \to \infty$.[5]

(ii) The non-renormalization statement for the SYM correlator is correct but there may be a subtlety in the definition of the O_8 operator, e.g., it may contain also $\frac{1}{N}\,\mathrm{tr}\, F^2\, \mathrm{tr}\, F^2$ admixture[6] (with various possible Lorentz contractions of the two F^2 factors). That would change the result for the three-point function on the SYM side and may make it zero.[7] However, this suggestion seems to be in contradiction with arguments based on $\mathcal{N} = 4$ supersymmetry [16, 29].

(iii) There may be subtleties in the supergravity calculation. The supergravity result may turn out to be non-vanishing and in agreement with the non-vanishing large-N SYM result.

In this paper we shall propose a natural resolution of this paradox based on the third option. The key observation is the following. Suppose that there was some

[4]That this correlator of the bosonic gauge-theory parts of O_4 and O_8 computed in the abelian theory limit is non-vanishing was already observed in the first paper in [25].

[5]One possibility could be that the $\mathcal{N} = 4$ supersymmetric expression for the 3-point function for the whole multiplet contains several tensor structures. However, this seems to be excluded by the arguments in [16, 17].

[6]We thank I. Klebanov and J. Polchinski for suggesting this possibility.

[7]The leading large-N term in $\langle (\mathrm{tr}\, F^2)^4 \rangle$ which is of order $O(1)$ comes from disconnected diagram, while the next order term (leading connected contribution) is of order $O(1/N^2)$. One can show (using the results of section 2 below) that $\langle O_4 O_4 O_8' \rangle = 0$ if $O_8' = O_8 - \frac{1}{4N}\,\mathrm{tr}\, F^2\, \mathrm{tr}\, F^2$.

non-vanishing two dilaton — fixed scalar coupling in the $D = 5$ action. Then the resulting 3-point function computed from supergravity according to the rules of [3] would actually be *divergent*. This can be seen from the general expression for the 3-point function of the scalar fields in the (euclidean) AdS_5 space found in [6]. If the scalar action is

$$S = \int d^5x\sqrt{g_5}\left[\frac{1}{2}\sum_{s=1}^{3}\left(\partial_m\varphi_s\partial^m\varphi_s + m_s^2\varphi_s^2\right) + \lambda_{123}\varphi_1\varphi_2\varphi_3\right], \tag{1.5}$$

then the corresponding 2- and 3-point correlators of the boundary operators are [6]

$$\langle O_\Delta(x_1)O_\Delta(x_2)\rangle = \frac{a_0}{|x_1-x_2|^{2\Delta}}, \qquad \text{with } a_0 = \frac{2(\Delta-2)}{\pi^2\Delta}\frac{\Gamma(\Delta+1)}{\Gamma(\Delta-2)}, \tag{1.6}$$

$$\langle O_{\Delta_1}(x_1)O_{\Delta_2}(x_2)O_{\Delta_3}(x_3)\rangle = \frac{\lambda_{123}\ a_1}{|x_1-x_2|^{\Delta_1+\Delta_2-\Delta_3}|x_1-x_3|^{\Delta_1+\Delta_3-\Delta_2}|x_3-x_2|^{\Delta_3+\Delta_2-\Delta_1}}, \tag{1.7}$$

where $\Delta_s = 2 + \sqrt{4 + m_s^2R^2}$ and[8]

$$a_1 = -\frac{\Gamma[\frac{1}{2}(\Delta_1+\Delta_2-\Delta_3)]\Gamma[\frac{1}{2}(\Delta_1+\Delta_3-\Delta_2)]\Gamma[\frac{1}{2}(\Delta_2+\Delta_3-\Delta_1)]\Gamma[\frac{1}{2}(\Delta_1+\Delta_2+\Delta_3-4)]}{2\pi^4\Gamma(\Delta_1-2)\Gamma(\Delta_2-2)\Gamma(\Delta_3-2)} \tag{1.8}$$

It follows from (1.8) that whenever the dimensions of the operators Δ_s satisfy the relation (or any of its two permutations)

$$\Delta_3 = \Delta_1 + \Delta_2 + n, \qquad n = 0, 1, 2, \dots, \tag{1.9}$$

the coefficient a_1 develops a pole. This is precisely what we encounter in the case of the $\phi\phi\nu$ interaction: since $m_\phi^2 = 0$ and $m_\nu^2 = 32/R^2$ one has $\Delta_1 = \Delta_2 = 4$, $\Delta_3 = 8$, i.e. $\Delta_3 = \Delta_1 + \Delta_2$. The final result is thus *undefined*: a product of a zero and a divergence.[9]

Our proposal is that in such special cases one should use a special regularization prescription in computing the supergravity expressions. One should effectively turn on "a little bit" of S^5 KK momentum for the dilaton. Then (i) the dilaton will get an infinitesimal mass ϵ, i.e. the dimension of O_4 will be shifted from 4 to $4+\epsilon$, thus regularizing (1.8), and (ii) the action will get a small coupling term $\epsilon\phi\phi\nu$. As a result, the dependence on ϵ will cancel out in the final expression in (1.7) ($\epsilon \times 1/\epsilon$=1) and we will obtain a finite result for $\langle O_4O_4O_8\rangle$. This procedure essentially amounts to an analytic continuation that treats the S^5-independent zero mode ϕ as an integral part of the whole family of S^5 KK modes of the 10-d dilaton field.

[8]Similar expression is found for any dimension d of the boundary: the numbers -2 and -4 are replaced in general by $-d/2$ and $-d$.

[9]If one regularizes the resulting IR divergence (e.g., by shifting the position of the boundary) the final result will be zero because of the vanishing of the bulk interaction constant.

Equivalently, we suggest that the AdS/CFT correspondence should be defined for the whole KK families of modes on the supergravity side, i.e. one is to compute, e.g., the supergravity $\phi\phi\nu$ correlator as a limit of the correlator for the two massive KK modes of the dilaton (with equal masses $m_k^2 = k(k+4)/R^2$, $k = 0, 1, 2, \ldots$) and ν. A non-zero KK quantum number k implies that now $\Delta_k = 4+k$ (the corresponding SYM operator is $O_{4+k} \sim \mathrm{Str}(F^2 X^k + \cdots))$ and thus the Γ-function factor in (1.7) $\Gamma[\frac{1}{2}(\Delta_1 + \Delta_2 - \Delta_3)]$ is simply $\Gamma(k)$. At the same time, the coupling constant $\lambda_{\phi\phi\nu}$ between ν and the two KK dilaton modes happens to be non-vanishing for $k \neq 0$ and is proportional to $k(k+4)/2^{k-1}(k+1)(k+2)$. As a result, the product of λ_{123} and a_1 in (1.7) is well-defined and finite ($\sim k\Gamma(k) = \Gamma(1+k)$) for *all* k*, including the case of* $k = 0$.

A similar prescription should be applied in all cases where the anomalous dimensions of the three operators in the correlation function satisfy the relation (1.9). With a hindsight, the suggested regularization procedure is actually implicit in the examples discussed in [10], where the 3-point functions $\langle \mathrm{tr}\, X^k \, \mathrm{tr}\, X^k \, \mathrm{tr}\, X^{2k} \rangle$ ($k = 2, 3, \ldots$) were shown to be finite and equal to their large-N (free-field) SYM counterparts. There the *poles* in the supergravity amplitudes (1.7) and (1.8) were similarly cancelled by *zeros* in the cubic lagrangean coupling derived from the KK reduction of the 10-d supergravity action.[10]

Close parallel with the case of the $\mathrm{tr}\, X^n$ 3-point functions and the supersymmetry relations between the corresponding operators on the SYM side (1.3) leave little doubt that the precise numerical AdS/CFT correspondence should then be found also in the present case of the normalized correlator $\langle O_4 O_4 O_8 \rangle / |\langle O_4 O_4 \rangle| |\langle O_8 O_8 \rangle|^{1/2}$ (moduli in the numerator denote the coefficients in the 2-point functions). To check this explicitly one is to compare the finite numerical coefficient obtained using our prescription on the supergravity side with the large-N SYM result. Since the comparison is done for the normalized correlator, one needs to know also the SYM expressions for $\langle O_4 O_4 \rangle$ and $\langle O_8 O_8 \rangle$. While the SYM computation of $\langle O_4 O_4 O_8 \rangle$ (discussed below) does not depend on the scalar and fermionic terms in the $\mathcal{N} = 4$ supersymmetric completion of the F^4 operator (1.2), to find $\langle O_8 O_8 \rangle$ one needs first to determine the full structure of O_8. This will be done in [31] where we will present the full expression for the $\mathcal{N} = 4$ super-invariant O_8 in terms of $\mathcal{N} = 1$ and $\mathcal{N} = 2$ superfields and compute $\langle O_8 O_8 \rangle$ using off-shell superfield formalism.

The plan of this paper is the following. In section 2 we shall present the SYM calculation for the correlator $\langle O_4 O_4 O_8 \rangle$ in the leading large-N (fixed $\lambda = g_{\mathrm{YM}}^2 N$) approximation. In section 3 we shall determine the supergravity expressions for the correlators $\langle O_4 O_4 O_8 \rangle$ using the "analytic continuation in KK momentum" prescrip-

[10] The observation about cancellation between poles and zeroes for special combinations of dimensions of the three operators in [10] was also made in ref. [30].

tion outlined above. Section 4 will contain some concluding remarks. In the appendix we shall discuss an attempt to determine the normalization of the O_8 operator from its coupling to the fixed scalar ν in the Born-Infeld D3-brane probe action.

2. Gauge theory calculation of $\langle O_4 O_4 O_8 \rangle$

In this section we shall compute $\langle O_4 O_4 O_8 \rangle$ correlator in large-N SYM theory and demonstrate that it is non-vanishing. As already mentioned above, one is able to find the precise expression for $\langle O_4 O_4 O_8 \rangle$ (at separated points, i.e. modulo contact terms) without knowing the detailed structure of the supersymmetric completion of F^4 terms in (1.2). It is easy to understand either from the action of supercharges on the chiral primary field (1.3) (see, e.g., [12]) or from the ($\mathcal{N} \geq 2$) superfield expressions [21] that the scalar and fermionic terms in O_4 are of the "equations of motion" form[11]

$$O_4 = \mathrm{tr}\left(F^2 + X\partial^2 X + \bar{\psi}\gamma \cdot \partial\psi + \cdots\right). \tag{2.1}$$

Contracted with other fields in the correlators these terms give rise only to contact contributions proportional to delta functions (which are field-redefinition dependent and are omitted in CFT correlation functions).[12]

As a result, the scalar and fermionic terms in O_4 and O_8 are irrelevant for the computation of $\langle O_4(x)O_4(0)\rangle$ and $\langle O_4(x)O_4(y)O_8(0)\rangle$ which can therefore be done simply in the pure YM theory. The correlator $\langle O_8(x)O_8(0)\rangle$ does, however, receive non-trivial scalar and spinor contributions, i.e. is different from the pure YM result [31].

We shall take the SU(N) Yang-Mills lagrangean in the form

$$\mathcal{L} = \frac{N}{4\lambda}\,\mathrm{tr}(F_{\mu\nu}F_{\mu\nu})\,, \tag{2.2}$$

[11] Here we are ignoring interaction (commutator $[X,X]$, $[\psi,X]$ dependent [12]) terms in O_4 since they produce terms of higher order in λ in the correlators (which altogether should cancel out in the cases where there is a non-renormalization theorem). Since the dilaton should be related to gauge coupling constant, one expects that the dilaton operator should be proportional to the SYM lagrangean, up to a total derivative term. The $\mathcal{N}=2$ supersymmetry implies that the quadratic scalar term should have the form $\mathrm{tr}(XD^2X)$, which is different from the standard kinetic term by a total derivative.

[12] This explains, in particular, the agreement between the result obtained using the dilatonic F^2 operator that follows from the BI action [23, 24] (in the Einstein frame the BI action gives $e^{-\phi}F^2 + (\partial X)^2 + \cdots$, i.e. scalars and fermions do not couple to ϕ) and the form of the corresponding $\mathcal{N}=2$ superinvariant that has the structure (2.1). The scalar and fermionic terms are important, however, in most of other relevant cases: for example, the stress tensor operator that couples to the 4-d graviton contains non-trivial scalar and fermionic terms [24] (see also [12]).

where $\lambda = g^2_{\rm YM} N$. We are assuming that the generators are normalized so that $\mathrm{tr}(T_i T_j) = \delta_{ij}$.[13] In the double-line notation, $A^a_{\mu b} = A^i_\mu (T_i)^a_b$, the gauge vector propagator is

$$\langle A^a_{\mu b}(x) A^c_{\nu d}(0)\rangle = \frac{\lambda}{N}\left(\delta^a_d \delta^c_b - \frac{1}{N}\delta^a_b\delta^c_d\right)\frac{\delta_{\mu\nu}}{4\pi^2 x^2}\,. \tag{2.3}$$

In the free-theory limit the field strength propagator is then

$$\langle F^a_{\mu\nu b}(x) F^c_{\lambda\rho d}(0)\rangle = \frac{\lambda}{N}\left(\delta^a_d \delta^c_b - \frac{1}{N}\delta^a_b\delta^c_d\right)\frac{2}{\pi^2 x^4} D_{\mu\nu,\lambda\rho}(x)\,, \tag{2.4}$$

where

$$D_{\mu\nu,\lambda\rho}(x) = \epsilon_{\mu\nu,\lambda\rho} - \frac{4}{x^2} X_{\mu\nu,\lambda\rho}\,, \tag{2.5}$$

and

$$\begin{aligned}
\epsilon_{\mu\nu,\lambda\rho} &\equiv \frac{1}{2}(\delta_{\mu\lambda}\delta_{\nu\rho} - \delta_{\mu\rho}\delta_{\nu\lambda})\,,\\
X_{\mu\nu,\lambda\rho} &\equiv \frac{1}{4}(x_\mu x_\lambda \delta_{\nu\rho} - x_\mu x_\rho \delta_{\nu\lambda} + x_\nu x_\rho \delta_{\mu\lambda} - x_\nu x_\lambda \delta_{\mu\rho})\,.
\end{aligned}$$

All correlators will be found in the free theory limit, i.e. ignoring all contributions which are sub-leading at large-N and small λ. Hence the field strength matrices in (1.1) and (1.2) do not contain the interaction term, i.e. are simply $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, and the computation of the correlators reduced to a straightforward application of (2.4). After lengthy calculations one finds[14]

$$\begin{aligned}
\langle O_4(x) O_4(0)\rangle &= \frac{48\lambda^2}{\pi^4}\frac{1}{x^8}\left(1 - \frac{1}{N^2}\right), & (2.6)\\
\langle O_4(x) O_4(y) O_8(0)\rangle &= \frac{1}{N}\frac{128\times 9\lambda^4}{\pi^8}\frac{1}{x^8 y^8} + O\left(\frac{1}{N^3}\right). & (2.7)
\end{aligned}$$

Similarly, one finds that the gauge-field part $\frac{3}{2}\,\mathrm{Str}[F^4 - \frac{1}{4}(F^2)^2]$ of the operator O_8 contribution to the correlator $\langle O_8(x) O_8(0)\rangle$ is

$$\langle O_8(x) O_8(0)\rangle_{\rm g.f.} = \frac{8\times 27\lambda^4}{\pi^8}\frac{1}{x^{16}} + O\left(\frac{1}{N^2}\right). \tag{2.8}$$

The contributions of the fermionic and scalar terms in O_8 increase the numerical coefficient in (2.8) [31].

[13] A more conventional choice of normalization of the generators in the fundamental representation is $\mathrm{tr}(T_i T_j) = \frac{1}{2}\delta_{ij}$, $\mathcal{L} = \frac{N}{2\lambda}\,\mathrm{tr}(F_{\mu\nu}F_{\mu\nu})$.

[14] In the case of abelian gauge theory these correlators were computed in [25]. The numerical coefficients in the correlators found in the abelian theory and the large-N non-abelian theory are *different*.

3. $D = 5$ supergravity results for the correlators

The type IIB supergravity action for the (Einstein-frame) metric, dilaton, RR scalar and 4-form is ($2\kappa_{10}^2 = (2\pi)^7 g_s^2 \alpha'^4$)

$$I_{10} = -\frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{g} \left[\mathcal{R} - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}e^{2\phi}(\partial C)^2 - \frac{1}{4 \cdot 5!}(F_5)^2 \right] . \qquad (3.1)$$

In what follows we shall concentrate on the dilaton — fixed scalar correlators, but the case where the dilaton is replaced by the RR scalar C (and the SYM operator tr F^2 is replaced by tr FF^*) is very similar. The background $AdS_5 \times S^5$ metric (which is an extremum of (3.1) [32]) is

$$ds_{10}^2 = R^2 \left[\frac{1}{z^2}(dz^2 + dx_\mu dx_\mu) + d\Omega_5^2 \right] , \qquad R^4 = 4\pi g_s N \alpha'^2 . \qquad (3.2)$$

In what follows we shall set the radius R to be 1. To dimensionally reduce to $D = 5$, we use the ansatz (1.4) for the 10-d Einstein frame metric. The relevant part of the type IIB supergravity action reduced to 5 dimensions for the zero modes of the dilaton and the metric is

$$I_5 = -\frac{1}{2\kappa_5^2} \int d^5x \sqrt{g_5} \left[\mathcal{R}_5 - \frac{1}{2}(\partial\phi)^2 - \frac{40}{3}(\partial\nu)^2 - V(\nu) + \cdots \right] , \qquad (3.3)$$

where[15]

$$V(\nu) = 8e^{-\frac{40}{3}\nu} - 20e^{-\frac{16}{3}\nu} = -12 + \frac{40}{3} \times 32\nu^2 + O(\nu^3) . \qquad (3.4)$$

This action does *not* contain ϕ–ν coupling (in particular, $\phi\phi\nu$ term) — this is a consequence of the fact that the dilaton does not couple to both $\mathcal{R}$ and $(F_5)^2$ in (3.1).

As was already explained in the introduction, our main suggestion is that to establish a correspondence between the SYM and the supergravity expressions for the $\langle O_4 O_4 O_8 \rangle$ correlator one should "regularize" the dimensional reduction procedure. One should first assume that ϕ has some "small" dependence on S^5 coordinates, leading to an infinitesimal 5-d dilaton mass term and an infinitesimal $\phi\phi\nu$ coupling, and take the corresponding small parameter to zero only after the computation of the $\phi\phi\nu$ correlator. The meaning of this prescription is that treating the zero mode of the dilaton on the same footing as higher KK modes resolves the ambiguity in the definition of the $\phi\phi\nu$ correlator and leads to a finite value for its overall numerical coefficient.

Let us start with decomposing ϕ in terms of spherical harmonics on S^5

$$\phi(y, x) = \sum_I Y^I(y)\phi^I(x) . \qquad (3.5)$$

[15] The two terms in the potential originate from the curvature of S^5 and the $(F_5)^2$ term.

We shall follow the notation of [10] and consider S^5 as embedded in a euclidean space R^6. The spherical harmonics Y^I are defined by $Y^I = C^I_{i_1 \dots i_k} y^{i_1} \cdots y^{i_k}$, where y^i $(i = 1, \dots, 6)$ are the cartesian coordinates of the embedding R^6 space $(y^i y^i = 1)$ and $C^I_{i_1 \dots i_k}$ are totally symmetric traceless tensors of rank k.

The $I = 0$ term in (3.5) $\phi^0(x) \equiv \phi(x)$ is the 5-dimensional dilaton while $\phi^I(x)$ are higher KK modes of the $D = 10$ dilaton. The dimensionally reduced form of the dilaton part of the action (3.1) is found to be

$$\frac{1}{2\kappa_{10}^2} \int d^{10}x \, \frac{1}{2} \sqrt{g} \, g^{MN} \partial_M \phi \partial_N \phi = \frac{1}{2\kappa_5^2} \int d^5x \sqrt{g_5} (L_1 + L_2) \,, \tag{3.6}$$

where $(\omega_5 = \pi^3)$

$$\frac{1}{2\kappa_5^2} = \frac{R^8 \omega_5}{2\kappa_{10}^2} = \frac{N^2}{8\pi^2} \,. \tag{3.7}$$

The two parts of the dilaton lagrangean are

$$L_1 = \frac{1}{2} g_5^{mn} \sum_I A(k) \partial_m \phi^I \partial_n \phi^I \,, \tag{3.8}$$

and

$$L_2 = \frac{1}{2} e^{-\frac{16}{3}\nu} \sum_I B(k) \phi^I \phi^I \,, \tag{3.9}$$

with the normalization constants $A(k)$ and $B(k)$ given by (k is the rank of $C^I_{i_1 \dots i_k}$)

$$A(k) = \frac{1}{2^{k-1}(k+1)(k+2)} \,, \qquad B(k) = k(k+4) A(k) \,. \tag{3.10}$$

The factors of e^ν in (1.4) where chosen so that the Einstein $\mathcal{R}$-term and hence L_1 have no ν-dependence. The form of the ν-dependent factor in L_2 (3.9) follows from the fact that $L_2 = e^{-\frac{10}{3}\nu(x)} \int_{S^5} g^{ss'} \partial_s \phi \partial_{s'} \phi$, where $g_{ss'} \sim e^{2\nu}$ is the S^5 part of the metric in (1.4).

Since for $I = 0$ one has $k = 0$ and thus vanishing $B(k)$, we conclude once again that there is no coupling between ν and the zero mode of the dilaton. Instead of taking $k = 0$ directly, one may formally set $k = \epsilon$ and take the limit $\epsilon \to 0$ only at the end of the calculation of correlation functions. As follows from (3.9), this prescription generates both a small mass for the 5-d dilaton and a coupling between the dilaton and ν,

$$m^2 = 4\epsilon \,, \qquad \lambda_{\phi\phi\nu} = -\frac{32}{3}\epsilon = -\frac{8}{3} m^2 \,. \tag{3.11}$$

Introducing the couplings between the AdS_5 boundary values of the 5-d fields ϕ and ν and the corresponding $\mathcal{N} = 4$ SYM F^2 and F^4 operators (1.1) and (1.2) as in [2, 3] we get

$$\int d^4x \, [a_4 \, O_4 \, \phi(x) + \, a_8 \, O_8 \, \nu(x)] \,, \tag{3.12}$$

where a_4 and a_8 are some normalization constants.

We learn from (3.3) and (3.12) that the operators O_4 and O_8 have the following dimensions ($m^2 \to 0$)

$$\Delta_4 = 2 + \sqrt{4 + m^2} = 4 + \frac{m^2}{4} + O(m^4)\,, \qquad \Delta_8 = 2 + \sqrt{4 + 32} = 8\,. \tag{3.13}$$

Following [2, 3] and using the expressions for the 2- and 3-point functions in [6] we find from (3.3) and (3.12) the following supergravity predictions for the correlators involving O_4 and O_8 (see eqs. (1.6)–(1.8))

$$\langle O_4(x) O_4(0)\rangle = \frac{1}{2\kappa_5^2}\frac{24}{\pi^2 a_4^2}\frac{1}{x^8}\,, \tag{3.14}$$

$$\langle O_8(x) O_8(0)\rangle = \frac{1}{2\kappa_5^2}\frac{42\times 24\times 40}{3\pi^2 a_8^2}\frac{1}{x^{16}}\,, \tag{3.15}$$

$$\langle O_4(x) O_4(y) O_8(0)\rangle = \frac{1}{2\kappa_5^2}\frac{8\times 36 m^2}{3\pi^4 a_4^2 a_8}\Gamma\left(\frac{m^2}{4}\right)\frac{1}{x^8 y^8} = \frac{1}{2\kappa_5^2}\frac{32\times 36}{3\pi^4 a_4^2 a_8}\frac{1}{x^8 y^8} + O(m^2)\,. \tag{3.16}$$

There is an extra factor of 2 in (3.16) coming from the fact that there are two dilaton field legs in the $\phi\phi\nu$ coupling (3.11). In (3.16) we have used that for $m^2 \to 0$, $\Gamma(m^2/4) \to 4/m^2$. The resulting expression for the $\phi\phi\nu$ correlation function is thus *finite*: the zero in the $\phi\phi\nu$ bulk interaction vertex cancels against the pole in the Γ-function coming from the integral over AdS_5.

Let us note in passing that there is also no $\phi\nu\nu$ coupling in (3.3), i.e.

$$\langle O_4(x) O_8(y) O_8(0)\rangle = 0\,. \tag{3.17}$$

Since the dimensions of the three operators here do not satisfy (1.9), in contrast to the case of $\langle O_4(x) O_4(y) O_8(0)\rangle$ the expression (1.8) for this correlator is well-defined and there is no ambiguity. The vanishing of $\langle O_4(x) O_8(y) O_8(0)\rangle$ deduced from the argument based on $\mathrm{U}(1)_Y$ charge conservation, see [19, 17]. From the supergravity point of view, this U(1) is part of $\mathrm{SL}(2,\mathbb{R})$ which is a classical symmetry of type IIB supergravity. The dilaton transforms non-trivially under this group, while ν is inert being part of the Einstein-frame metric. Thus the coupling like $\phi\nu\nu$ is forbidden in the classical supergravity lagrangean.

Since the supergravity expressions (3.14)–(3.16) depend on the normalization constants appearing in (3.12), the direct comparison with SYM result (2.7) is not possible. The weighted three-point function does not depend on the absolute normalization,

$$\langle \mathcal{O}_4(x) \mathcal{O}_4(y) \mathcal{O}_8(0)\rangle = \frac{1}{N}\sqrt{\frac{16}{105}}\frac{1}{x^8 y^8}\,, \tag{3.18}$$

where $\mathcal{O}_4$ and $\mathcal{O}_8$ are rescaled O_4 and O_8 satisfying

$$\langle \mathcal{O}_4(x) \mathcal{O}_4(0)\rangle = \frac{1}{x^8}\,, \qquad \langle \mathcal{O}_8(x) \mathcal{O}_8(0)\rangle = \frac{1}{x^{16}}\,. \tag{3.19}$$

To compare (3.18) with the SYM result we need the expression for $\langle O_8 O_8 \rangle$ in SYM theory which requires a detailed separate computation and will be presented in [31]. Here we note only that the value of $\langle O_8 O_8 \rangle$ in SYM theory which would lead to the agreement with (3.18) is

$$\langle O_8(x) O_8(0) \rangle = \frac{105 \times 36}{\pi^8} \frac{\lambda^4}{x^{16}} \,. \tag{3.20}$$

Not surprisingly, this expression is different from the pure gauge theory part (2.8) of the full SYM correlator.

One heuristic way to fix the values of the normalization constants a_4 and a_8 in (3.12) is to derive (3.12) from the BI action for a D3-brane propagating in a curved background. As shown in the appendix this leads to

$$a_4 = \frac{N}{4\lambda} \,, \qquad a_8 = \frac{10 N \pi^2}{9 \lambda^2} \,, \tag{3.21}$$

where $\lambda = g_{\rm YM}^2 N = 4\pi g_s N$. The leading large-$N$ term in (2.6) is in perfect agreement with the supergravity expression (3.14) with a_4 given by the BI normalization in (3.21) (this agreement is essentially the one originally found in [23, 24, 2]). If we use a_8 from (3.21) in (3.15) we find that the supergravity expression for $\langle O_8(x) O_8(0) \rangle$ is

$$\langle O_8(x) O_8(0) \rangle_{\rm BI} = \frac{84 \times 81 \lambda^4}{5\pi^8} \frac{1}{x^{16}} \,, \tag{3.22}$$

and from (3.16) we get

$$\langle O_4(x) O_4(y) O_8(0) \rangle_{\rm BI} = \frac{1}{N} \frac{96 \times 36 \lambda^4}{5\pi^8} \frac{1}{x^8 y^8} \,. \tag{3.23}$$

The numerical coefficient in (3.23) is different from the one in (2.7) by factor of 3/5. This disagreement may be related to ambiguities in using the BI action for fixing the normalization of operators beyond the leading second-derivative terms (see the appendix).

4. Concluding remarks

What follows from our proposal is that though some of the couplings in the S^5 compactified 5-d (supergravity + KK multiplets) action may appear as missing they are actually 'hidden' and must be taken into account in establishing the AdS/CFT correspondence. This may carry an important message about how the AdS/CFT correspondence is actually working. The 10-d picture with non-zero KK momentum seems to be more fundamental than the 5-d one. This is after all what the original precise conjecture [1] about the duality of *ten-dimensional* string on $AdS_5 \times S^5$ and $\mathcal{N} = 4$ SYM theory is implying.

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On the supergravity/string side the 5-d fields appear as KK modes of $D = 10$ fields upon compactification on S^5. The families of 5-d fields originating from a single type IIB supergravity field should definitely 'remember' their common 10-dimensional origin. At the same time, this 10-dimensional origin of the corresponding composite operators on the SYM side is quite obscure. The $AdS_5 \times S^5$ string — SYM correspondence suggests that there may exist a '10-dimensional' reformulation of $\mathcal{N} = 4$ SYM theory where these operators are somehow unified in 'KK families'.

One of our motivations for trying to get a detailed understanding of how the AdS/CFT correspondence works at the level of 3-point functions is to shed some light on the factorization properties of much more complicated 4-point correlators. In the previous discussions of the supergravity expression for the 4-point function of the 'dilatonic' O_4 operator [28, 33, 34, 35], the contribution of the scalar ν exchange to the four-dilaton amplitude was not included. This may be justified by noting that although the 3-point amplitudes (1.7) are divergent for the operators satisfying (1.9), the exchange amplitude contributing to the 4-point function $\langle O_{\Delta_1} O_{\Delta_1} O_{\Delta_2} O_{\Delta_2} \rangle$ with the exchanged state corresponding to the operator O_{Δ_3} is actually finite [36, 30]. Naively, one may conclude that the ν-exchange contribution to the four-dilaton amplitude is indeed zero. This seems to provide an example when the existence of a non-vanishing three-point function ($\langle O_4 O_4 O_8 \rangle$) does not imply the presence of the corresponding factorization contribution to the 4-point ($\langle O_4 O_4 O_4 O_4 \rangle$) correlator.

Though the supergravity integral representing contribution of fixed scalar is convergent and is multiplied by zero couplings in the $k \to 0$ limit, the story may be more intricate. The supergravity expression for the 4-dilaton amplitude could be defined as a $k \to 0$ limit of the amplitude $\langle O_{4+k} O_{4+k} O_{4+k} O_{4+k} \rangle$ for four massive ($m_k^2 = k(k+4)/R^2$) dilaton modes. This amplitude receives contributions from both graviton and fixed scalar exchanges. It may happen that when the correlator $\langle O_4 O_4 O_4 O_4 \rangle$ is defined in this way, its factorization will become more transparent (cf. [33, 36, 30, 35]).

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Note added. Since this paper was written, another paper [37] appeared which confirmed our proposal and made it explicit that the option (iii) from the introduction is realized.

A. Normalizations of O_4 and O_8 operators from Born-Infeld action

Here we describe an attempt to fix normalizations in (3.12) by starting with a Born-Infeld action.[16] The Born-Infeld lagrangean for (a collection of) D3-branes in curved background has the form:

$$\mathcal{L} = T_3 \ \mathrm{Str} \sqrt{\det(\hat{g}_{\mu\nu} + e^{-\frac{\phi}{2}} \mathcal{F}_{\mu\nu})} + \ \mathrm{WZ-terms}\,, \tag{A.1}$$

where

$$\mathcal{F}_{\mu\nu} = 2\pi\alpha' F_{\mu\nu}\,, \qquad T_3 = \frac{1}{(2\pi)^3 g_s \alpha'^2}\,. \tag{A.2}$$

We take the background metric Einstein-frame metric as a combination of (3.2) and (1.4):

$$ds^2 = R^2 \left[e^{-\frac{10}{3}\nu} \frac{dz^2 + dx_\mu dx_\mu}{z^2} + e^{2\nu} d\Omega_5^2 \right], \tag{A.3}$$

$$R^4 = \lambda\alpha'^2\,, \qquad \lambda = 4\pi g_s N = g^2_{\mathrm{YM}} N\,. \tag{A.4}$$

Then the relevant part of the lagrangean in (A.1) becomes

$$\begin{aligned}
\mathcal{L} &= T_3 \left(\frac{R^2}{z^2} e^{-\frac{10}{3}\nu} \right)^2 \mathrm{Str} \left(\left[\det(\delta_{\mu\nu} + e^{-\frac{\phi}{2}} \frac{z^2}{R^2} e^{\frac{10}{3}\nu} \mathcal{F}_{\mu\nu}) \right]^{1/2} - 1 \right) \\
&= T_3 \left(\frac{R^2}{z^2} e^{-\frac{10}{3}\nu} \right)^2 \mathrm{Str} \left[\frac{1}{4} e^{-\phi} \left(\frac{z^2}{R^2} e^{\frac{10}{3}\nu} \right)^2 \mathcal{F}^2 - \frac{1}{8} e^{-2\phi} \left(\frac{z^2}{R^2} e^{\frac{10}{3}\nu} \right)^4 \left(\mathcal{F}^4 - \frac{1}{4} (\mathcal{F}^2)^2 \right) + \cdots \right] \\
&= \frac{1}{4} T_3 e^{-\phi} (2\pi\alpha')^2 O_4 - \frac{1}{8} T_3 e^{-2\phi} \left(\frac{z^2}{R^2} e^{\frac{10}{3}\nu} \right)^2 (2\pi\alpha')^4 \frac{2}{3} O_8 + \cdots \\
&= \cdots - \left(\frac{N}{4\lambda} \phi O_4 + \frac{10 N \pi^2}{9\lambda^2} \nu z^4 O_8 \right) + \frac{N\pi^2}{3\lambda^2} \phi z^4 O_8 + \cdots\,.
\end{aligned} \tag{A.5}$$

Here we used the definitions of O_4 and O_8 in (1.1) and (1.2). Comparison with (3.12) then implies[17]

$$a_4 = \frac{N}{4\lambda}\,, \qquad a_8 = \frac{10 N \pi^2}{9\lambda^2}\,. \tag{A.6}$$

The normalization of O_8 derived in this way is only heuristic and should be treated with caution.

Indeed, this argument may look suspicious for several reasons. First, ν already couples to scalars X at the second derivative level but $(\partial X)^2$ cannot be part of the O_8

[16] Similar approach of fixing normalizations from BI action was used in [24, 38].

[17] In taking the boundary limit the factor z^4 produces a cutoff factor which should multiply F^4 on dimensional grounds.

operator. Also, ν interacts *differently* with F^4 and $(\partial X)^4$ terms, in contradiction with the fact that they must be parts of the same super-invariant. The scalar partners of the gauge field couple to ν at quartic order as $e^{4\nu}(\partial X)^4$, which leads to the normalization coefficient different from the one in (A.6) — since the coefficient of ν, 10/3, is replaced by 2, we get

$$a'_8 = \frac{2N\pi^2}{3\lambda^2}\,. \tag{A.7}$$

Remarkably, this is *precisely* the coefficient one needs for the correspondence between the supergravity and SYM expressions for the 3-point functions (see eqs. (3.16) and (2.7)). However, due to the ambiguities mentioned above it seems hard at the moment to make this argument convincing.

In general, the BI action [39, 40, 41] summarizes the low-energy limit of scattering amplitudes of massless open string modes in flat space. Its direct curved-space generalization correctly accounts for certain zero-momentum limits of the disc amplitudes with few closed string vertex operator insertions (see, e.g., [42]). One should keep in mind, however, that the standard BI action does not include terms with derivatives of the vector field strength, higher than the first derivative of the scalar (coordinate) fields and terms with derivatives of the closed string fields. As a result, there may be a clash with manifest (linearly realized) $\mathcal{N} \leq 4$ supersymmetry: the manifestly supersymmetric extensions of the structures appearing in the expansion of the bosonic BI action may involve higher derivatives; elimination of such higher-derivative terms via integration by parts may produce terms with derivatives of the background supergravity fields, i.e. terms which are not included in the standard BI action. In particular, one should not be surprised if one does not find the correct bosonic (scalar) parts of the corresponding superinvariants from the expansion of the BI action.

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